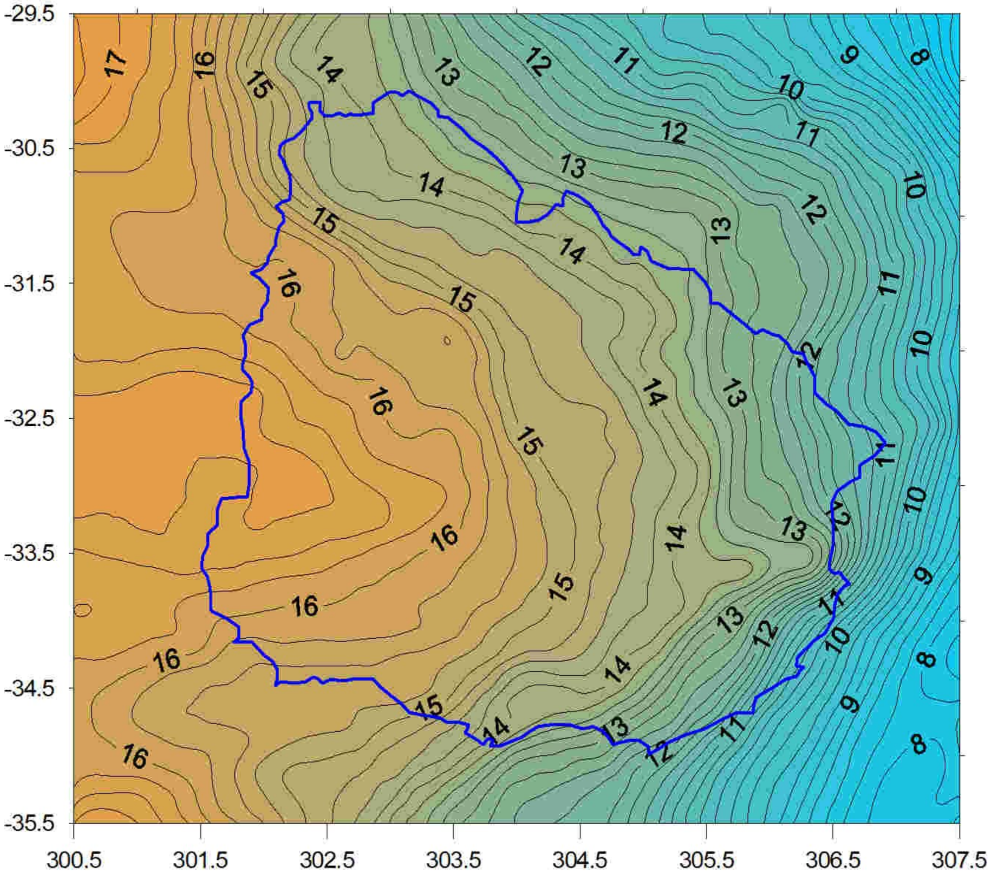
UruGeoide110 (2023)
Technical Report



Calculation of the UruGeoide110 (2023)

The objective of calculating a centimetre geoid depends greatly on obtaining the minor frequencies of the geoid, which can only be extracted from a high resolution MDT and/or a very dense gravimetry, as it exists in some European countries. Taking this into consideration, for the calculation of the UruGeoide110 model (2023), a different strategy was adopted from that used in previous models, in which the processing was carried out above the entire area but the resolution of the MDTs used had decreased based on the available computational programs and media.

Thus, with the purpose of using the new land model of Uruguay, measured with Lidar technology, (2.5 m spatial resolution) and after having made a first calculation with upper spatial resolution, it was decided that the base resolution would be of 90 m and divide the total area in 4 blocks. This took into account the total extension of the project area and the processing capacity of existing programs and computers, as well as the possibility of taking advantage of the original data of the SRTM90 model. Also to achieve effective processing, it was decided to process the blocks of land and gravimetry data in a first instance, and then integrate the results for the final calculation. The size of each block was determined based on previous experiences (including the calculation of the preliminary UruGeoide2022a[1]), and the structure and conformation of each defined block is detailed below.

1- Preparation of land and gravity data

a- Digital Land Model

The project area (7° φ x 8° λ) was divided into 4 processing blocks of 4° x 4.5°, which were called T1 to T4, thus obtaining an overlap of 0.5° with each adjacent block. For the creation of the MDT, digital surface models of the IDE for Uruguay, SRTM90 (V2) were used for the continental part of Argentina and Brazil and the DTU18 model for the bathymetry of the oceanic part. The IDE model has a spatial resolution of 2.5 m, SRTM90 of 90 m and DTU18 of 1800 m. Figure 1 shows the 4 blocks with the aforementioned overlap.

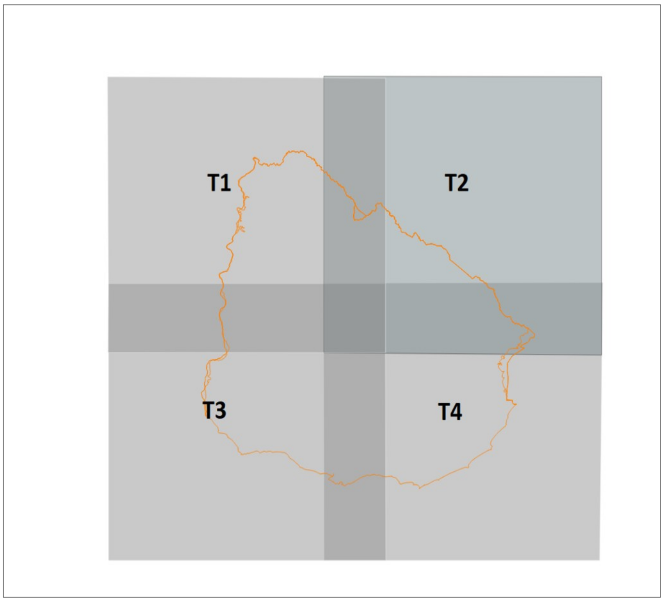


Figure 1 – Block scheme for land

To extract the land data, two cutting polygons were created for the image files of the digital models (TIF), in order to obtain an overlap of 0.1° between the digital data of the IDE model and the SRTM data. The blue exterior polygon serves as a cut for the IDE model and the green inner polygon for SRTM data. Figure 2 presents the aforementioned polygons.

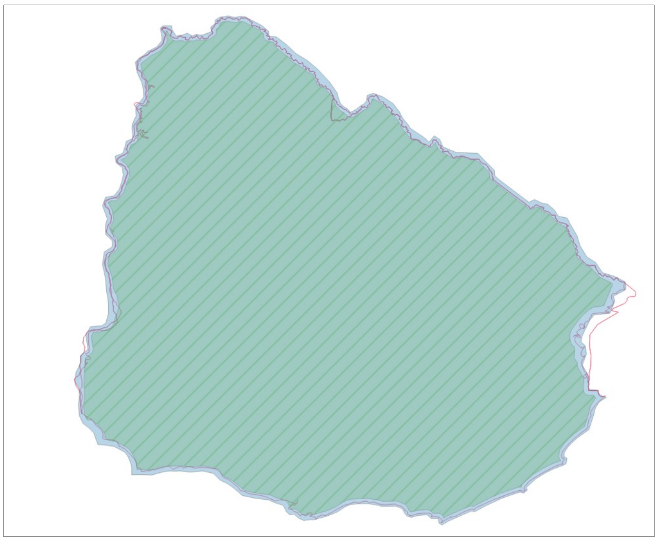


Figure 2 - Cutting polygons for land

Both, the original SRTM model and the IDE model, were integrated into a resolution of 90 m or 0.008333° and the geodetic system of each one was transformed for Sirgas2000. SRTM model cutting example in block 1 (T1), with overlap of the interior area of Uruguay, covered by the IDE model (Figure 3).

Figure 3 - SRTM model cut in block 1

IDE model cutting example in block 1 (T1) (Figure 4).

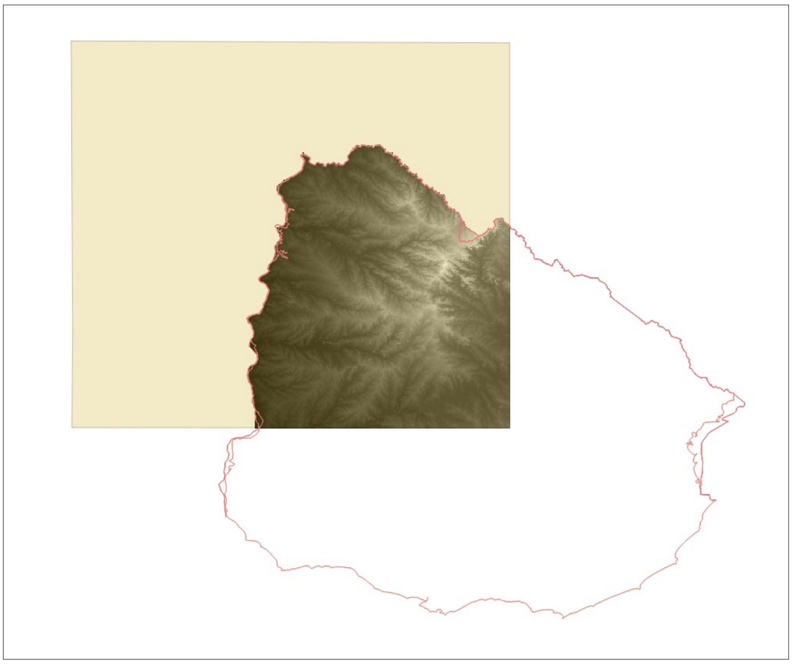


Figure 4- IDE model cut (90) in block 1

Detail of the integration of the two models on the northeast border of Uruguay, department of Artigas. (Figure 5)

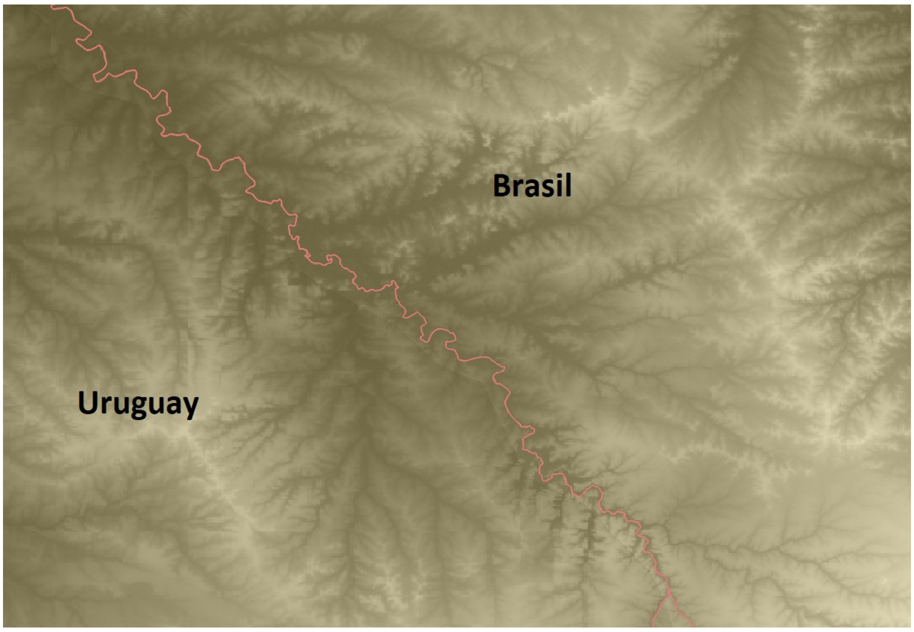


Figure 5 - Integration of SRTM and IDE data on the northern border

In the next step, the files of each block 1 to 4 of land were transformed into XYZ files in order to obtain text files with coordinates and altitudes. These files were purified of points that had no altitude values (Nodata Points) and integrated with bathymetric values obtained from the DTU18 model. The final points file thus formed was interpolated with a spatial resolution of 180 m or 6”, creating a MDT file in grid format for each processing block. This resolution of 180 m was the one that was best found, for the purpose of not generating points without altitude or nodata values. Example of interpolation of block T1 (Figure 6).

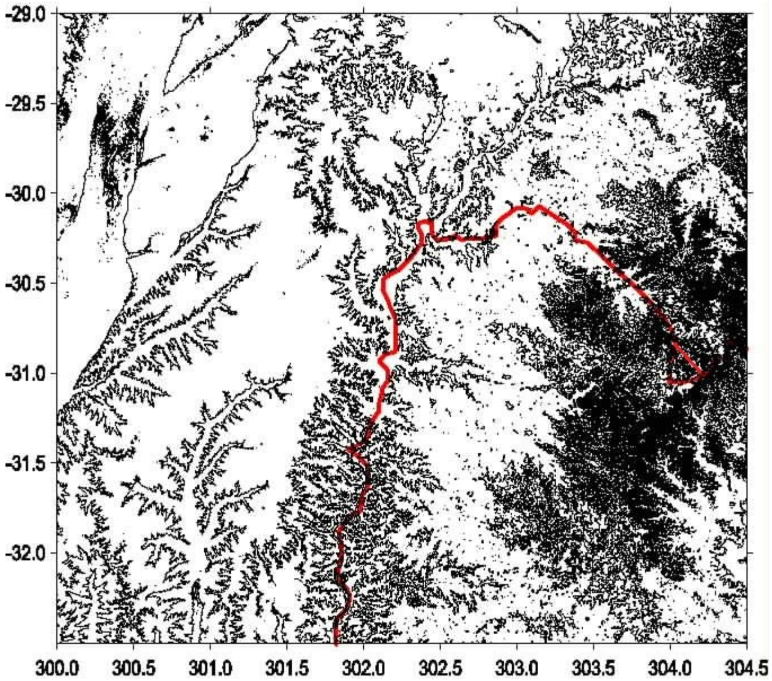


Figure 6 - MDT for block 1

Integration of the 4 blocks and DTM obtained with level curves (Figure 7).

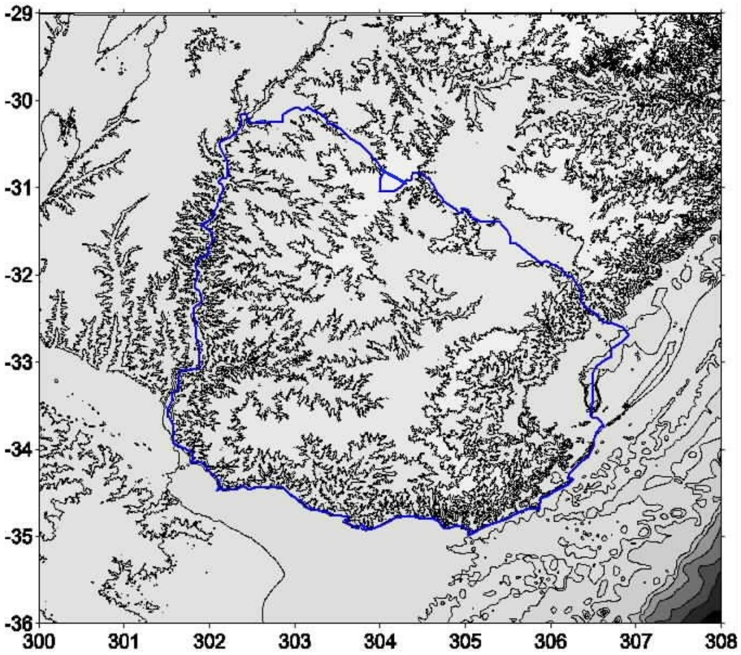


Figure 7 – DTM for the entire project area

b- Complementary land

From the created model with a resolution of 180 m or 6” two derived models were created, for the calculation of the gravimetric effect of land on gravity anomalies and in geoidal undulations. The first with a resolution of 900 m or 30”, and the second one a reference grid (altitudes reference plane to be used with the Residual Terrain Model - RTM), with a resolution of 3600 m or 120”. To obtain an optimal smooth of the reference plane, two different operations were carried out in sequence: first it makes an average grid of 4×4 cells (3.6 km x 3.6 km) and then the grid obtained with windowed with an average operator that moves into the entire area with a 28×28 cell window (approx 25 km x 25 km).

Figures 8 and 9 show the results for block 1: the 60”, and the reference model respectively.

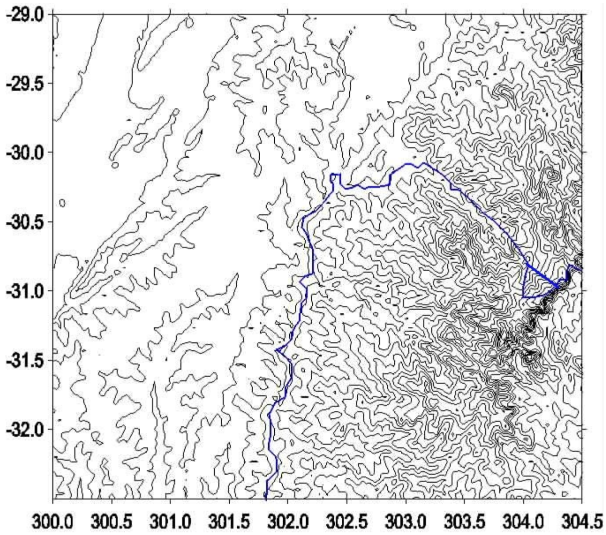


Figura 8 – 60” modelo

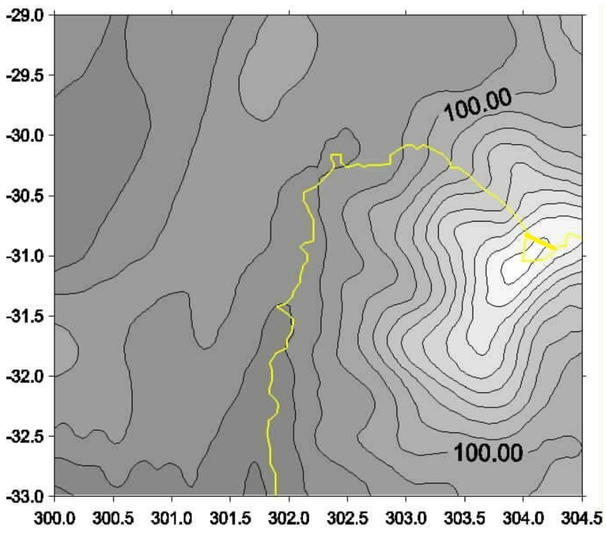


Figura 9 – 120” reference model

The mentioned procedure was executed in all processing blocks.

c- Dados de gravimetría

The gravimetric data of Uruguay was adjusted in 1995[2], and recently again in 2022, using 3 absolute stations. The gravimetry data of Argentina, Brazil and Uruguay were integrated into a database, were processed to obtain free air outdoor and Bouguer anomalies. The geodetic reference system used was GRS80, with application of atmospheric correction and transformation of gravity for "tidal free" system (free tide). The total land gravimetric data was 10429 and 10089 free air anomalies data in oceanic areas, based on the DTU13 model. The geopotential model used for gravity anomalies was EIGEN-6C-4 to degree and order of 720 (approximate resolution of 28 km).

Next, 4 interior polygons were created to the terrain polygons, called G1 to G4, with a reduction of 0.5 ° on each side and an overlap between them 0.2°, so that you can select the gravimetric data that would be processed by each block of DTM With this we guarantee that all the gravity of each G block, would be processed with sufficient land information throughout its extension and there will be redundancy of gravimetric data.

The 4 processing blocks were configured as follows (Figure 10):

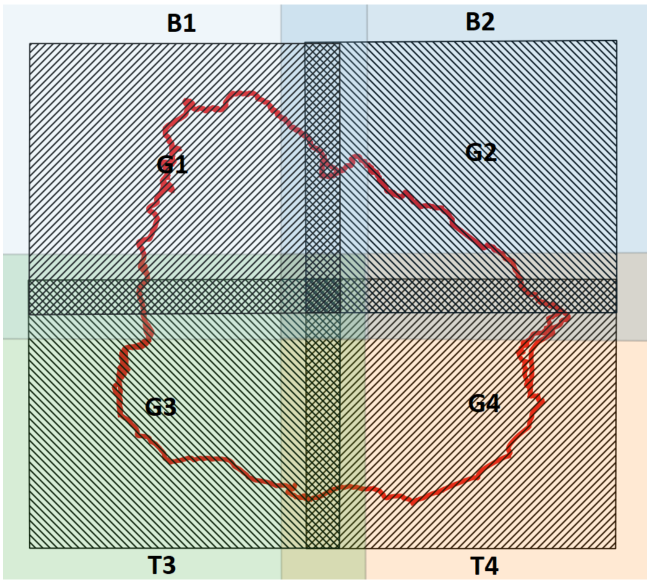


Figure 10 - Processing Blocks

The space coverage of each of the mentioned blocks is detailed in Table 1 and the amount of data selected in each polygon in Table 2.

Table 1- Space extension of each gravity and DTM block

BLOCK	Polygon	Min Lat	Max Lat	Min Lon	Max Lon	Δ Lat	Δ Lon
	TOTAL AREA	-36,0	-29,0	300,0	308,0	7,0	8,0
1	Gravity - G1	-32,7	-29,5	300,5	304,2	3,2	3,7
	DTM- T1	-33,0	29,0	300,0	304,5	4,0	4,5
2	Gravity – G2	-32,7	-29,5	303,8	307,5	3,2	3,7
	DTM- T2	-33,0	-29,0	303,5	308,0	4,0	4,5
3	Gravity – G3	-35,5	-32,3	300,5	304,2	3,2	3,7
	DTM- T3	-36,0	-32,0	300,0	304,5	4,0	4,5
4	Gravity– G4	35,5	32,3	303,8	307,5	-3,2	3,7
	DTM- T4	-36,0	-32,0	303,5	308,0	4,0	4,5

Table 2- Files and quantity of data in each processing block

	DTM		Gravity	
Block	Files	Quantity	Files	Quantity
1	T1_int_90.dat	9916956	G1_freeair.dat	3491
	T1_ext_90.dat	18092780		
	T1_bat_180.dat	6375	G1_bouguer.dat	
	Total – T1_final.dat	**28016111**		
2	T2_uy_90.dat	7878374	G2_freeair.dat	2773
	T2_ext_90.dat	19597637		
	T2_bat_180.dat	266354	G2_bouguer.dat	
	Total – T2_final.dat	**27742365**		
3	T3_uy_90.dat	10480886	G3_freeair.dat	3175
	T3_ext_90.dat	12138946		
	T3_bat_90.dat	1625974	G3_bouguer.dat	
	Total – T3_final.dat	**24245806**		
4	T4_uy_90.dat	12070137	G4_freeair.dat	5143
	T4_ext_90.dat	2841234		
	T4_bat_90.dat	3414524	G4_bouguer.dat	
	Total – T4_final.dat	**18325895**		
	Total	**98330177**		**14582**

The result of the MDT block selection and gravity is presented in Figure 12 for block 1. Note the outer coverage of the MDT for the processing of red gravity data.

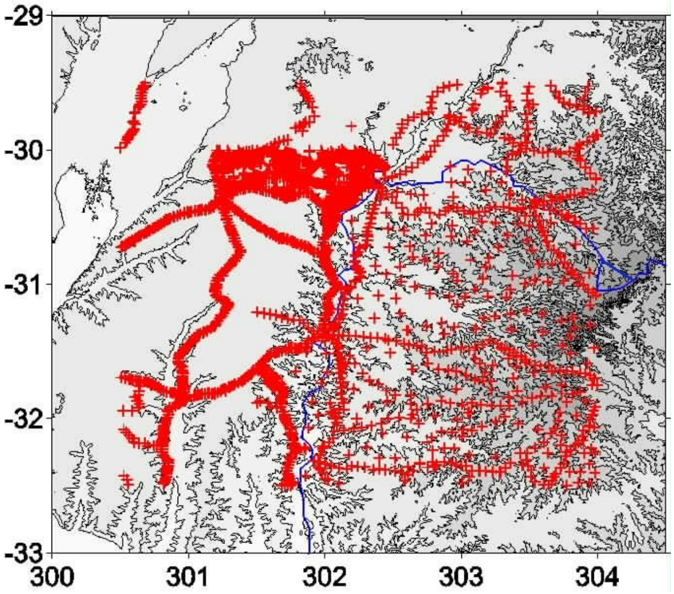


Figure 12 - Block 1 with DTM and gravity data

2- Geoid calculation

a- Block processing (see Annex 1)

The initial calculation is done separately in each block (see Annex 1), all the used programs belong to the Gravsoft geophysical package - Geodetic Gravity Field Modelling Program[2]. First, gravity anomalies are reduced from the gravity derived from the chosen geopotential model, thus obtaining a first reduced anomaly (Geoip program). Secondly, from the reduced anomalies of the previous step, the terrain gravimetric effect is removed, resulting in anomalies with complete reduction (geopotential and land model) (TC[3] program). The next block calculation refers to the residual effect of terrain in height anomalies by the RTM method. The results of each block are integrated into a unique file for subsequent creation of a residual land effects (TC program). The process from this point is carried out for the entire project area.

b- Processing for the whole project Area

Previous works with all processed blocks (see Annex 2)

- Bouguer's anomalies are integrated into a single file with the purpose of transforming the quasi-geoid calculated into a geoidal model at the end of the calculation process.
- The anomalies with complete reduction are integrated into a unique file, for creation of a grid of reduced anomalies.
- The 30” calculated grids are integrated into a file that is interrapted for the entire area, with a resolution of 60”.
- The calculated RTM terrain effects are integrated into a file and interpolated for the entire project area.

Proceso (ver Anexo 3)

The anomalies with complete reduction obtained previously are gridded (**GEOGRID** program) and using the Stokes formula with Fast Fourier Transform (FFT) the height anomalies are calculated (**SPFOUR** program).
To the result, the RTM terrain effects already calculated (**GCOMB** Program) and the contribution of the geopotential model in geoidal undulations (**GEOIP** Program) are added.
With these operations we obtain the quasi-geoid.

The next step is to transform the quasi-geoid model into a geoid model, using Bouguer's anomalies and the 60” DTM model.
First we transform the Bouguer anomalies´ file into a grid format (**GEOGRID** program) and then the difference between quasi-geoid and geoid is calculated using the 60” DTM model, using the formula (**GCOMB** Program).

$$\zeta - N = H_P - H_P^N \approx -\frac{g_P - \gamma_P + 0{,}1967(mGal/m)H}{\gamma_0} = -\frac{\Delta g_{Boug}}{\gamma_0} H$$

The calculated differences are added to the quasi-geoid, obtaining the final gravimetric geoid (**GCOMB** **p**rogram).
The detailed processing of the calculation is found in Annex 1 to 3. Figure 13 shows the final result for the gravimetric geoid, called UruGeoide110 (2023).

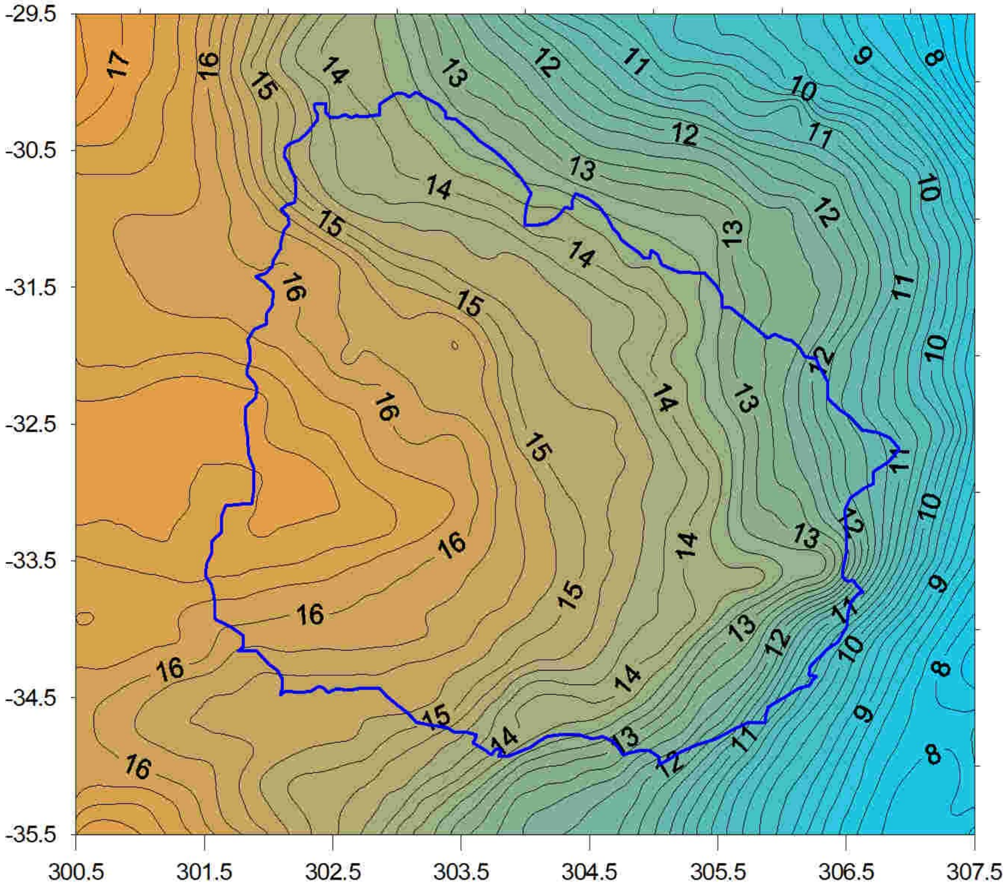


Figure 13 – UruGeoide110 (2023)

Other considered corrections

In addition to the transformation of the observed gravity for free-tide tide system, the correction of the zero order term was calculated. This correction is applied by the difference between the geodetic and geopotential systems of the global model used and the local data system.

In the first case, the difference in gravity between the mean tide and the tidal-free system was found between a minimum of -10 and a maximum +2×10-8 m/s-^{2} (precision level of the absolute gravimeter JILAg -3).

The calculation of the zero order term for geoidal undulations was carried out according to the formula in Sánchez *et al* [3]:

$$N_0 = \frac{\left(GM_{EIGEN} \; - \; GM_{GRS80}\right)}{\left(r_{P0} \; * \; \gamma_{Q0}\right)} \; - \; \frac{\left(W_{0\,EIGEN} \; - \; U_{0\,GRS80}\right)}{\left(\gamma_{Q0}\right)}$$

Replacing with the values of the geopotential model EIGEN-6-C4 and the GRS80 reference geodesic system:

$$N_0 = \frac{\left(5{,}85\,x\,10^7\,m^3 s^{-2}\right)}{\left(r_{P0} \; * \; \gamma_{Q0}\right)} \; - \; \frac{\left(9{,}14\,x\,10^7\,m^2 s^{-2}\right)}{\left(\gamma_{Q0}\right)} \approx -0.005 \quad a \;\; -0.004\,m$$

Both results, of tidal system and zero order term, were considered non-significant in the present state of the geoidal calculation.

3- Transformation model for Datum Cabildo (see Annex 4)

The geoidal model is of the gravimetric type for its input and calculation form data through the Stokes formula and for its practical use, it must be adapted to the official Uruguayan datum, Cabildo. This adaptation involves distortion caused by errors inherent to the establishment and measurement of the country's vertical network. It is in that sense that we call the geoidal model adapted to the vertical network of *transformation model* and not geoid.

To calculate the transformation model, we need stations measured with GNSS that also have altitude in the local vertical system, thus having an observed geoidal undulation. Of the total available stations, some are chosen, with the best possible spatial distribution and the remaining ones are left as comparison stations to find what the error we would have in any transformation made.

For this model we have 96 stations, from which 51 were separated for the transformation of the geoid and 45 for comparison and evaluation.

The final model detailed in Annex 4, presented a level difference with the Datum Cabildo of +78 cm and a standard deviation of 5 cm. In relation to the control stations, the average difference is 1 cm with a 7 cm s.d. Figure 14 shows the transformation stations in red and those of control in orange, finally Figure 15 shows the transformation model, called IGM110_Cabildo.

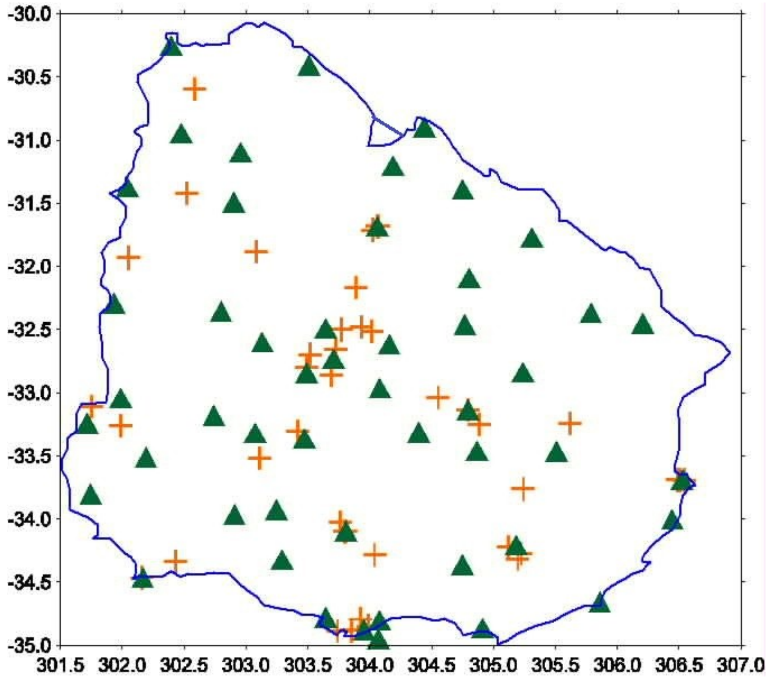


Figure 14 - Transformation and comparison stations

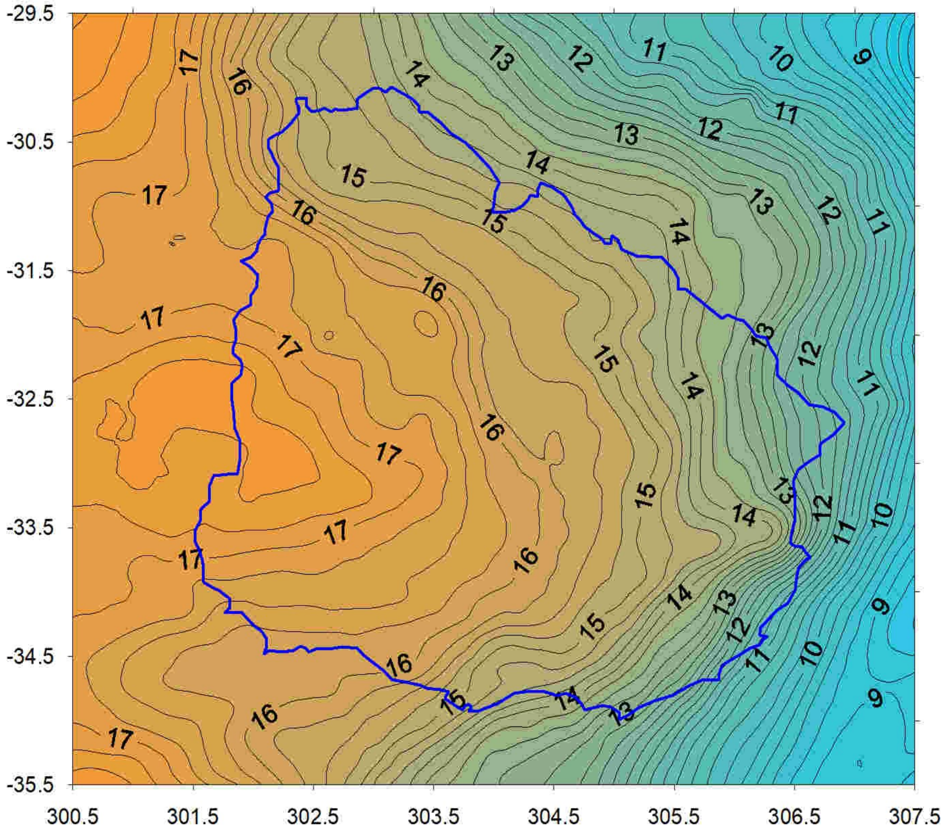


Figure 15 - IGM110_Cabildo transformation model

The calculated transformation model seems to carry out errors of determination and measurement of GNSS stations on points of the vertical network. To improve the precision of the model, the measurement of points of the first or second order vertical network that is being adjusted in terms of geopotential numbers is necessary.

We thanks the National Geographic Institute of Argentina (IGN Ar), the Brazilian Institute of Geography and History (IBGE) and the University of Sao Paulo (Cenegeo), Brazil for its generous contribution of data that allowed this project. Likewise, the Denmark Space Agency is appreciated for the assignment of the programs and models used, as well as the Prof René Forsberg for its orientation and to the Prof Ludger Timmen of the Leibnitz University of Hannover (IFE), Germany for its continued support in the gravimetry area. All this support is gratefully acknowledged.

References

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4- Forsberg, R. y Tscherning C.C., GRAVSOFT - Geodetic Gravity Field Modelling Programs 3r,d edition, August 2008/August 2015.

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Annex 1

URUGEOIDE110 (2023) 90m

Processing by Blocks

PROGRAM	Block	Stage, entry files and results	mean	std.dev.	min	max

1-Gravimetric anomalies are reduced from the geopotential model to degree and order of 720.

$\Delta g^{rd1} = \Delta g^{al} - \Delta g^{egm}$

PROGRAM	Block	Stage, entry files and results	mean	std.dev.	min	max
GEOIP (job 1a)	Block 1	original data (G1_airelibre.dat) – 3491	7,99	9,40	-74,15	52,90
		grid interpolation results (EIGEN_al_720_25.gri)	8,03	8,21	-13,26	47,92
		predicted values output (B1_al.rd1)	-0,04	4,38	-83,52	18,77
	Block 2	original data (G2_airelibre.dat) – 2773	23,13	20,16	-44,36	96,81
		grid interpolation results (EIGEN_al_720_25.gri)	18,76	14,22	-25,14	46,12
		predicted values output (B2_al.rd1)	4,37	10,42	-50,29	108,46
	Block 3	original data (G3_airelibre.dat) – 3175	12,74	11,14	-17,98	118,28
		grid interpolation results (EIGEN_al_720_25.gri)	12,67	8,89	-11,91	40,35
		predicted values output (B3_al.rd1)	0,07	5,48	-19,03	93,20
	Block 4	original data (G4_airelibre.dat) – 5143	10,82	15,77	-17,33	100,40
		grid interpolation results (EIGEN_al_720_25.gri)	10,14	15,26	-14,41	83,08
		predicted values output (B4_al.rd1)	0,68	5,29	-62,49	35,00

2- Calculate RTM terrain effects and subtract from anomalies

$\Delta g^{rd} = (\Delta g^{al} - \Delta g^{egm}) - \Delta g^{H}$

PROGRAM	Block	Stage, entry files and results	mean	std.dev.	min	max
TC (job 1b)	Block 1	difference given - dtm inferred station heights	-6	11	-100	10
		computed effects, no of points: 3491	-0,40	2,78	-11,83	21,69
		original values in statfile (B1_al.rd1)	-0,04	4,38	-83,52	18,77
		difference output on file (B1_al.rd)	0,36	4,20	-83,18	19,20
	Block 2	difference given - dtm inferred station heights	4	18	-195	411
		computed effects, no of points: 2773	1,82	8,42	-30,54	34,81
		original values in statfile (B2_al.rd1)	4,37	10,42	-50,29	108,46
		difference output on file (B2_al.rd)	2,55	9,65	-32,49	115,14
	Block 3	difference given - dtm inferred station heights	3	5	-25	43
		computed effects, no of points: 3175	0,07	1,79	-4,39	11,91
		original values in statfile (B3_al.rd1)	0,07	5,48	-19,03	93,20
		difference output on file (B3_al.rd)	0,00	5,36	-19,36	91,29
	Block 4	difference given - dtm inferred station heights	29	39	-53	634
		computed effects, no of points: 5143	2,49	5,24	-10,44	39,75
		original values in statfile (B3_al.rd1)	0,68	5,29	-62,49	35,00
		difference output on file (B3_al.rd)	-1,81	6,89	-64,11	33,36

3- Calculation of topography´s effect of on height anomalies

ζ^{H}

PROGRAM	Block	Stage, entry files and results	mean	std.dev.	min	max
TC (job 1c)	Block 1	difference given - dtm inferred station heights:	-87	58	-399	1
		statistics of computed effects, no of points: 37800	-0,01	0,04	-0,07	0,19
	Block 2	difference given - dtm inferred station heights:	-166	118	-760	40
		statistics of computed effects, no of points: 37800	0,01	0,12	-0,30	0,43
	Block 3	difference given - dtm inferred station heights:	-45	47	-267	30
		statistics of computed effects, no of points: 65311	0,01	0,03	-0,06	0,14
	Block 4	difference given - dtm inferred station heights:	21	309	-460	2940
		statistics of computed effects, no of points: 65311	-0,01	0,13	-0,99	0,33

Annex 2

Processing for the entire project area

PROGRAM	**Preparation of gravity and land data**	
	Bouguer anomalies file composition	Quant.
	G1_Bouguer.dat	3491
	G2_Bouguer.dat	2773
	G3_Bouguer.dat	3175
	G4_Bouguer.dat	5143
textpad	**B1234_al.rd**	**14582**

	60 ” land file composition from 30” files	Quant.
	T1_30 481 x 541	260221
	T2_30 481 x 541	260221
	T3_30 481 x 541	260221
	T4_30 481 x 541	260221
textpad	**T1234_30** **point file**	**1040884**
uru.job	**T1234_60.gri** **421 x 481** **gridded file**	**202501**

	Composition of the reduced anomalies file	Quant.
	B1_al.rd	3491
	B2_al.rd	2773
	B3_al.rd	3175
	B4_al.rd	5143
textpad	**B1234_al.rd**	**14582**

	Composition of the indirect effect on height anomalies	Quant.
surfer	**B1_z_rtm.dat**	**65311**
surfer	**B2_z_rtm.dat**	**65311**
surfer	**B3_z_rtm.dat**	**65311**
surfer	**B4_z_rtm.dat**	**65311**
textpad	**B1234_z_rtm.dat** **point file**	**261224**
cuad_z_rtm.job	**B1234_z_rtm.gri** **421 x 481** **gridded file**	**202501**

Annex 3

1- Calculation of quasi-geoid (height anomalies)

a- Grid reduced anomalies

PROGRAM	Δg^{rd}	mean	std.dev.	min	max
GEOGRID (job 2a)	Predicted: 151981 points	-0,16	6,09	-40,92	85,10
	prediction error values			0,23	6,76

b- Stokes through spherical FFT (obtaining anomalies of reduced height)

	$\Delta \mathbf{g}^{rd\ cuad} \rightarrow \zeta^{RED}$	min	max	R.M.S	max abs
SPFOUR (job 2a)	Minimal and maximal values	-0,40	0,65		
	value of innerzone correction			0,01	0,08

c – Restore the undulations of the geopotential model (ζ^{MG})

We add the reduced height anomalies to RTM terrain height anomalies

	$\zeta^{RED} + \zeta^{h}$	mean	std.dev.	min	max
GCOMB (job 2b)	B1234_z_rd1.gri - reduced height anomalies	0,00	0,08	-0,40	0,65
	B1234_z_rtm.gri - RTM terrain height anomalies	0,00	0,01	-0,06	0,07
	B1234_z_rd_rtm1.gri - RTM reduced height anomalies	0,00	0,07	-0,34	0,62

Restore the geopotential model and obtain the **quasigeoid**

	$\zeta^{MOD} = \zeta^{RED} + \zeta^{H} + \zeta^{MG}$	mean	std.dev.	min	max
GEOIP (job 2b)	original data B1234_z_rd rtm1.gri	0,00	0,07	-0,34	0,62
	grid interpolation results eigen_n_720_25.gri	13,72	2,51	6,84	17,39
	predicted values output B1234_z1.gri QUASIGEOID	13,72	2,51	6,93	17,39

2) Quasi-geoid transformation in geoid

a) grid the point file Bouguer anomalies

		mean	std.dev.	min	max
GEOGRID (job 2c)	Prediction 151981 points – G1234_anom_Bou.gri	4,55	14,02	-76,37	103,12
	prediction error values			0,23	12,05

b) Calculate the difference between geoidal heights and height anomalies

	$\Delta^{N}{}_{Z}$	mean	std.dev.	min	max
GCOMB (job 2c)	G1234_anom_Bou.gri	4,55	14,02	-76,37	103,12
	T1234_60.gri	83,98	89,27	-717,96	585,42
	difHz.gri	0,00	0,00	-0,05	0,01

c) add the differences between geoidal heights and height anomalies

	$N^{MOD} = \zeta^{MOD} + \Delta^{NZ}$	mean	std.dev.	min	max
GCOMB (job 2c)	B1234_z2.gri	13,72	2,51	6,93	17,39
	difHz.gri	0,00	0,00	-0,05	0,01
	B1234_n1.gri GRAVIMETRIC GEOID	13,72	2,51	6,93	17,39

Annex 4

Height transformation model for Datum Cabildo

1) Find the differences between the geoidal model and the control file

PROGRAM		mean	std.dev.	min	max
GEOIP (job urugeo3)	original data (uru_gps_2022.n) 51 observations N^{gps}	15,39	1,51	11,34	17,69
	grid interpolation results – B1234_n1.gri N^{interp}	14,62	1,52	10,57	16,98
	predicted values output – dif_n_gps1 Δ^{NZ}	0,78	0,13	0,30	1,03

2) Grid the differences

	$\Delta^{NZ} \rightarrow \Delta^{NZ\ cuad}$	mean	std.dev.	min	max
GEOGRID (job urugeo3)	prediction pts 151981 points – dif_n_gps1.gri	0,78	0,05	0,66	0,88
	prediction error values			0,06	0,13

Four parameters trend transformation solution

detrending done on data, itrend =	5			
no of trend parameters estimated:	4			
solution:	5,548	-6,080	-4,479	-8,489
detrended data (min,max,mean,stddev):	-0,47	0,24	0,00	0,13

3) Add the differences to the geoid and obtain the IGM110_Cabildo transformation model

	$\mathbf{N^{MOD} + \Delta^{NZ\ cuad} = uru^{modtransf}}$	mean	std.dev.	min	max
GCOMB (job urugeo3)	Gravimetric geoid – B1234_n1.gri N^{MOD}	13,72	2,51	6,93	17,39
	Differences – dif_n_gps1.gri $\mathbf{\Delta^{NZ\ cuad}}$	**0,78**	**0,05**	**0,66**	**0,88**
	Transformation model – B1234_n_corr.gri $uru^{modtransf}$	14,49	2,48	7,78	18,11

4) Differences between the transformation model and comparison stations

		mean	std.dev.	min	max
GEOIP (job compar)	original data - control.n , 45 comparison stations	15,48	1,18	12,38	17,53
	grid interpolation results	15,47	1,16	12,42	17,51
	predicted values output - dif_B1234_n_corr_control.dat	**0,01**	**0,07**	**-0,18**	**0,22**